Journal of Engineering Mathematics **32:** 19–26, 1997.

Lateral spreading of fibre tows

S.D.R. WILSON
Department of Mathematics, University of Manchester, Oxford Road, Manchester M13 9PL, U.K.

Received 12 September 1995; accepted in revised form 30 September 1996

Abstract. In the manufacture of fibre-reinforced polymer composite materials, the standard technology involves an intermediate product, 'prepreg'. To make this, the fibres are unwound from a reel in the form of a continuous bundle, or tow, impregnated with the resin matrix and spread out into a thin flat tape. The fibre tow is pulled through a bath of molten resin, passing over a series of bars in zig-zag fashion. The bars serve to locate the tow, control the tension, spread it laterally (and simultaneously reduce its thickness) and squeeze the resin in. The paper presents a simple theory of this spreading and thinning process, giving the spread width w in terms of the cross-section area A of the tow and the lateral offset of the spreader bar H by the simple explicit formula $w = (12AH)^{1/3}$. Simple experiments to check the theory are reported. The theory also gives an estimate of the thickness, which is more important in the impregnation process, but not readily accessible to direct measurement.

Key words: fibre tow, spreader bars, pultrusion, prepreg

1. Introduction

In the manufacture of fibre-reinforced polymer composite materials, an important preliminary stage involves the impregnation of a bundle of fibres (known as a 'tow') with liquid resin. A general review of this 'prepreg' technology has been given by Wiedemann and Rothe [1]. A commonly-used technique, known as pultrusion, involves pulling the fibre tow through a bath of liquid resin; the bath contains a number of cylindrical pins or bars fixed transversely to the general direction of motion of the tow, and also displaced laterally from this direction; the tow passes over the bars in zig-zag fashion. This process has been discussed and modelled by Chandler *et al.* [2] in some detail. As the tow passes over each bar, liquid resin is dragged in and forced into the spaces between the fibres by the pressure created by the longitudinal tension in the tow.

This pumping action is the main purpose of the pins; but they will also cause the tow to spread laterally. As the tow spreads laterally it will obviously get thinner and this will presumably help the impregnation process. Sometimes spreader bars are introduced for just this reason. Although the tow thickness appears in the model developed by Chandler *et al.* [2], no attempt is made to estimate it either from theory or measurement. Instead they choose a value which gives the best overall fit between theoretical and experimental estimates of the increase in line tension.

The purpose of the present paper is to derive a simple theoretical estimate of the lateral spreading of a fibre tow caused by pulling it over a bar. In the first place we consider a symmetrical arrangement with one spreader bar as sketched in Figure 1. The theory predicts that the width of the tow will take the value

$$w = (12AH)^{1/3}, \tag{1}$$

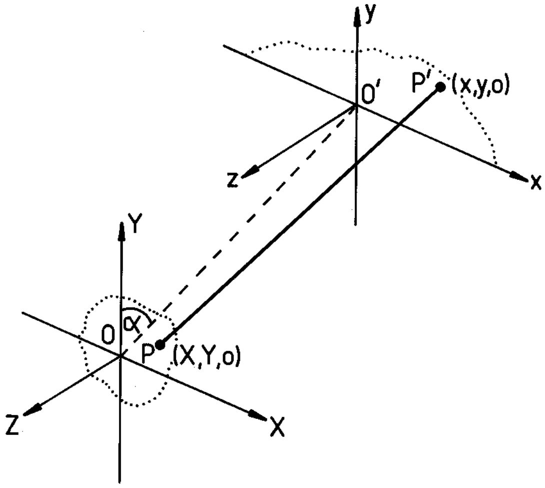

Figure 1. Sketch of (half of) the symmetrical one-bar spreader arrangement. The tow is gripped at O and is pulled symmetrically over a spreader bar, modelled as a thin blade whose upper edge coincides with $O'x$. PP' represents a typical fibre.

where A is the cross-section area of the tow and H is the lateral displacement of the bar; and also gives a prediction of the thickness. This result depends on a number of simplifying assumptions which will be explained in detail in the next section.

Following that, we describe the results of a series of simple experiments to test Equation (1). The results are in reasonable agreement, with a systematic error of about 10–15% for which a reasonable explanation is available.

Finally we extend the theory to cover the case of a symmetrical arrangement including several spreader bars.

2. The model

We begin by considering a symmetrical one-bar arrangement of which one half is sketched in Figure 1. The bundle of fibres is held at O, passes over a spreader bar, here considered as a thin knife edge lying along $O'x$, and then is gathered symmetrically. We set up two coordinate systems $OXYZ$ and $O'xyz$ as shown. The planes OXY and $O'xy$ are parallel and $OO' = L$. The angle $O'OY$ is α and then, referred to the frame $OXYZ, O'$ has coordinates $(0, L\cos\alpha, -L\ \sin\alpha)$. The quantity $H = L\cos\alpha$ has been referred to as the 'lateral displacement' in the previous section.

Now consider a typical fibre PP' which crosses the OXY plane at P and the $O'xy$ plane at P'. Again referring to the frame $OXYZ$, we observe that P has coordinates (X, Y, O) and P' has coordinates $(x, y + L\cos\alpha, -L\ \sin\alpha)$ and so the vector $\underline{PP'}$ is given by

$$\underline{PP'} = (x - X, y - Y + L\cos\alpha, -L\ \sin\alpha). \tag{2}$$

Suppose that the tension in the fibre is T. Then we can calculate the components of the force exerted at P', in the plane $O'xy$. The x component is $2T(X - x)/d$ and the y component is $2T(Y - y - L\cos\alpha)/d$, where

$$d = |\underline{PP'}|. \tag{3}$$

We assume that the arrangements are such that $x, y, X, Y \ll L$. In fact we shall assume that X and Y have a length scale $a \ll L$; then a is the 'diameter' of the original tow. The length scales for x and y will be determined as part of the theory. Then the expressions for the force components just given can be approximated by:

$$x - \text{component} : 2T(X - x)/L,$$

$$y - \text{component} : 2T\left[-\cos\alpha + \frac{Y - y}{L}\ \sin^2\alpha\right].$$

Next we assume that the number of fibres is very large and that the aggregate of the fibres in the plane $O'xy$ can be modelled as a continuum. The tension resultants can be thought of as being balanced by a hydrostatic pressure $p(x, y)$ satisfying

$$\frac{\partial p}{\partial x} = \frac{2Tn}{L\ell}(X - x), \tag{4}$$

$$\frac{\partial p}{\partial y} = \frac{2Tn}{L\ell}(-L\cos\alpha + (Y - y)\ \sin^2\alpha). \tag{5}$$

Here n is the number of fibres per unit cross-section area, and ℓ is the (short) distance in the z direction over which the tension acts, that is, the width of the spreader bar. Additional assumptions made here are that each fibre has the same tension, and that friction between fibres and between fibres and the spreader bar have no important resultants in the cross-section plane. The model described here does not of course incorporate any motion of the fibres along their length. This is considered to be steady and will have no dynamical effect, but in practice it will have the effect of 'shaking down' the fibres and so the neglect of friction is probably reasonably good. Also neglected are any effects of twisting or entanglement; each fibre is 'free to move'. In the actual pultrusion process efforts are made to ensure that the tows are in fact not twisted or tangled, since that would clearly impede the impregnation process. As regards the assumption of uniform tension, this is usually done, and it is difficult to see how any other assumption would lead to a workable theory. In fact, the fibre tows are unreeled and, when impregnated, reeled in at prescribed speed, the tension having increased by friction over the impregnation bars. Since these end conditions involve essentially the application of equal displacements to every fibre at the ends, the tension should be more or less uniform.

Next we obtain the appropriate dimensionless version of Equations (4) and (5). As noted we assume that the scale for X and Y is a. There is a natural small parameter ϵ given by

$$\epsilon = (a/L)^{1/3} \tag{6}$$

and it is easily shown by consideration of the orders of magnitude of the terms in Equation (4) and (5) that the scales for x and y are

$$x \sim a/\epsilon, \qquad y \sim a\epsilon \tag{7}$$

and the scale for p is $2Tna\epsilon/\ell$. Then we obtain

$$\frac{\partial p}{\partial x} = -x + \epsilon X, \tag{8}$$

$$\frac{\partial p}{\partial y} = -\cos\alpha + \epsilon^3 Y \sin^2\alpha - \epsilon^4 y \sin^2\alpha, \tag{9}$$

using the same letters for the dimensionless variables.

If we neglect all the small terms, we obtain immediately

$$\begin{aligned} p &= -\tfrac{1}{2}x^2 - y\cos\alpha + \text{const} \\ &= -\tfrac{1}{2}x^2 - y\cos\alpha + \tfrac{1}{2}c^2, \end{aligned} \tag{10}$$

say. The boundary condition is that

$$p = 0 \quad \text{on} \quad y = y_s(x), \tag{11}$$

where $y_s(x)$ is the shape of the free surface, which is to be determined. So

$$y_s(x)\cos\alpha = \tfrac{1}{2}(c^2 - x^2). \tag{12}$$

We can find the constant c by requiring that the total cross-section area equals 1, and thus

$$c = (\tfrac{3}{2}\cos\alpha)^{1/3}. \tag{13}$$

Notice that the shape of the tow at the spreader bar is independent of the value of T. The situation here is analogous to that which would arise when we consider the equilibrium shape of a liquid mass under the influence of a body-force field. This shape is independent of the magnitude of the force field; that is, the equipotential surfaces would be the same if the field were multiplied by any (scalar) constant, other than zero, when the problem becomes indeterminate. The shape is also independent of any details in the OXY plane. So the initial shape of the tow is irrelevant to this order of approximation.

Translating Equation (13) back into physical variables, we find that the width of the tow, w, is given by

$$w = (12\cos\alpha L a^2)^{1/3} = (12AH)^{1/3}, \tag{14}$$

where A is the cross-section area and $H = L\cos\alpha$ is the lateral displacement. This result will be tested by experiment. We obtain the dimensionless thickness at the centre by putting $x = 0$ in Equation (12).

3. Experiments

The carbon and glass fibres used in the industrial process are fragile and somewhat hazardous, so two kinds of fibres used in the textile industry (kindly supplied by the Textile Engineering Department, U.M.I.S.T.) were used. One was a continuous filament tow of polyamide fibres ‘Courtelle’), consisting of about 10^5 fibres, each of diameter about 23 microns. The other was a tow of cotton yarn consisting of about 600–700 threads. Each thread was itself a twisted yarn of much thinner microfibres; the thickness of each thread therefore was somewhat indeterminate as they extended under tension.

Both specimens had to have considerable weights attached to straighten and compress them laterally. The filaments of the polyamide tow had a permanent crimp, apparently for ease of

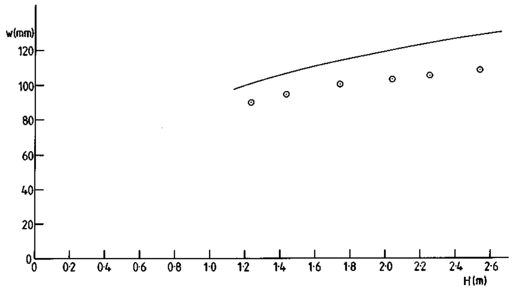

Figure 2. Comparison of theory (line) and experiment (points) for the continuous filament (Courtelle) tow. The cross-section area was approximately 70 mm^2.

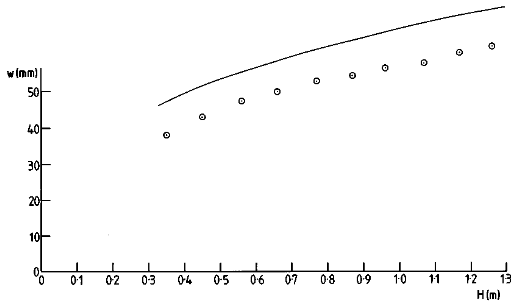

Figure 3. Similar comparison for the cotton yarn. The cross section area was 26 mm^2.

handling, and the cotton threads consisted of twisted yarn as noted. The slight ‘compressibility’ of the fibres, resulting from their composite structure, was the source of some error in the measurement of w and of the cross-section area A, since both would change slightly if the attached weight was varied. The actual weight used, about 30 kg, was about the largest which would be convenient and safe; the results were reproducible and it is not thought that much further compression would have been possible (but see below). In any case the compressibility effect will not arise in the case of monofilament fibres, so there seems little point in attempting to modify the theory to account for it.

The experimental procedure consisted simply of anchoring the tow at one end, passing it over a spreader bar (a rod of diameter about 12 mm), and then over a second bar placed so as to give symmetry, and then to a weight to keep the fibres straight and close-packed. The various bars were fitted into a steel frame with pre-drilled holes (used for another purpose) and kindly made available by the Department of Civil Engineering, Manchester University.

The lateral displacement H was measured with a tape measure, and the width of the tow with a pair of dividers. The latter measurement was subject to about 3–4% error because of stray fibres at the edges.

In order to compare with Equation (14) it was necessary to measure A and this proved somewhat troublesome. Eventually the tows were pulled over a steel strip with a rectangular notch, in place of the spreader bar, and area calculated from the measured width and depth.

The results are shown in Figures 2 and 3. The experimental points are the mean of 4 observations. The theory is Equation (14).

For one series of experiments the spreader bar was replaced by a piece of 1 mm sheet steel bent into an inverted V shape with a radius of curvature at the apex of about $2-3$ mm. This had no noticeable effect on the results; and we might conclude that the modelling of the spreader bar as a thin blade, in the previous section, will give good results.

As noted, there is a systematic difference between theory and experiment of 10–15%. This may be due to friction or entanglement. There was no naked-eye evidence of serious entanglement, but friction was certainly important. We could reach a range of equilibrium values of w by spreading or squeezing the tow by hand at the spreader bar. We made the actual observations after passing it back and forth a few times until it appeared to settle down. In addition, the value of A is presumabably a slight overestimate because of compressibility, as noted.

The general trend of the results appears good and, in particular, the cube-root dependence on H appears robust.

4. Further theory

It is clear that pulling the tow over several spreader bars will spread the tow out even further, but we might also expect these further widenings to be comparatively slight. We can investigate this by returning to Figure 1, but now suppose that the OXY plane is located at the first spreader bar and the $O'xy$ plane at the second. Then the cross section in the OXY plane has already been stretched out according to the length scales given in Equation (7). If we repeat the calculations leading up to Equation (7), we find that the scales for x and X are the same, and those for y and Y are the same, namely

$$x, X \sim a/\epsilon, \qquad y, Y \sim a\epsilon \tag{15}$$

and Equations (8) and (9) are replaced by

$$\frac{\partial p}{\partial x} = -x + X + O(\epsilon^3), \tag{16}$$

$$\frac{\partial p}{\partial y} = -\cos\alpha + O(\epsilon^4). \tag{17}$$

A difficulty is now created by the mixture of coordinate systems and we recast the equations in terms of X and Y. This has the advantage that the initial shape of the tow is supposedly given and so the domain of the equations is known.

Suppose the fibre at (X, Y) passes through (x, y), where

$$x = u(X, Y), \qquad y = v(X, Y). \tag{18}$$

The incompressibility condition can be written

$$\frac{\partial(u, v)}{\partial(X, Y)} = \frac{\partial u}{\partial X}\frac{\partial v}{\partial Y} - \frac{\partial u}{\partial Y}\frac{\partial v}{\partial X} = 1. \tag{19}$$

We can write

$$\frac{\partial p}{\partial x} = \frac{\partial(p, v)}{\partial(X, Y)}, \qquad \frac{\partial p}{\partial y} = -\frac{\partial(p, u)}{\partial(X, Y)} \tag{20}$$

with the help of equation (19), and after a short calculation we obtain

$$\frac{\partial p}{\partial X} = -(u - X)\frac{\partial u}{\partial X} - \cos\alpha\frac{\partial v}{\partial X}, \tag{21}$$

$$\frac{\partial p}{\partial Y} = -(u - X)\frac{\partial u}{\partial Y} - \cos\alpha\frac{\partial v}{\partial X}, \tag{22}$$

and these, with Equation (19), comprise the somewhat obscure equations which determine p, u and v. However we note that, by eliminating p from Equations (21) and (22), we find

$$\frac{\partial u}{\partial Y} = 0, \quad u = u(X), \tag{23}$$

which tells us that vertical columns remain vertical and just get altered in length and moved sideways. From Equations (19) we then get

$$\frac{\partial v}{\partial Y} = \frac{1}{u'(X)}, \qquad v = \frac{Y}{u'(X)}, \tag{24}$$

since $v = 0$ on $Y = 0$.

Next we find from Equation (22)

$$p = -\cos\alpha v + A(X), \tag{25}$$

where $A(X)$ is arbitrary. If we suppose that the profile of the tow in the (X, Y) plane is given by

$$Y = f(X), \tag{26}$$

we can determine $A(X)$ from the condition $p = 0$ on $Y = f(X)$. Thus

$$p = \frac{\cos\alpha}{u'(X)}(f(X) - Y). \tag{27}$$

Finally we use Equation (21), with the formulas just derived for for v and p, to obtain an equation for $u(X)$. This is

$$\frac{\cos\alpha}{u'(X)} f(X) + \int_0^X (u - X)u'(X)\,\mathrm{d}X + K = 0. \tag{28}$$

There is also a regularity condition. The function $f(X)$ will vanish at $X = 1$ (the edge of the tow), but we cannot have $u'(X) = 0$ at $X = 1$ or the material would pile up into a singularity. Hence

$$\int_0^1 (u - X)u'(X)\,\mathrm{d}X + K = 0. \tag{29}$$

The problem is still nonlinear of course and looks intractable in general. However, we can find an exact solution in the case

$$f(X) = \tfrac{3}{4}(1 - X^2), \tag{30}$$

which is a parabolic curve enclosing unit area with the X axis. This is particularly useful in view of Equation (12). The solution is

$$u = \lambda X, \tag{31}$$

where the constant λ is given by

$$\lambda^2(\lambda - 1) = \tfrac{3}{2}\cos\alpha. \tag{32}$$

(This satisfies Equations (28) and (29) as can be easily verified.)

The width of the tow in the $O'xy$ plane is $u(1) = \lambda$. This is greater than 1, but not in order of magnitude, as was suggested earlier.

This theory has been sketched on the assumption that the tow passes above both bars. It may well pass (say) under the first and over the second, and the theory then needs to be adjusted. We still have Equation (23), but Equation (24) is altered to

$$v = \frac{Y - f(X)}{u'}, \tag{33}$$

since $v = 0$ on $Y = f(X)$, and in place of Equation (27) we find

$$p = -\cos\alpha \frac{Y}{u'}, \tag{34}$$

because $p = 0$ on $Y = 0$. The exact solution given by Equations (30)–(32) is recovered when $f(X)$ is replaced by its negative.

5. Concluding remarks

A simple theory has been developed to estimate the width of a bundle or tow of fibres when it is pulled symmetrically over a spreader bar. The result, given in Equation (14), depends only on the cross-section area of the tow and the lateral displacement of the spreader bar, and not on the tension or the details of how the tow is gripped at its end.

This result, which probably contains all the information of importance in the engineering application, has been satisfactorily corroborated by experiment.

A further development of the theory provides estimates of the (smaller) effects of additional spreader bars.

Acknowledgement

The author would like to thank Dr A.F. Jones for his contribution to the theory.

References

1. G. Wiedemann and H. Rothe, Review of prepreg technology. In: G. Pritchard (ed.), *Developments in Reinforced Plastics, 3*. London: Applied Science Publishers (1982) pp. 83–119.
2. H.W. Chandler, B.J. Devlin and A.G. Gibson. A model for the continuous inpregnation of fibre tows in resin baths with pins. *Plastics, Rubber and Composites Processing and Applications* 18 (1992) 215–220.